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Cite as: AIP Conference Proceedings **1895**, 050012 (2017); https://doi.org/10.1063/1.5007384
Published Online: 12 October 2017

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Algorithm for Solving of Two-level Hierarchical Minimax Program Control Problem of Final State the Regional Socio-economic System in the Presence of Risks

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Abstract. In this paper we study the problem of optimization of guaranteed result for program control by the final state of regional social and economic system in the presence of risks. For this problem we propose a mathematical model in the form of two-level hierarchical minimax program control problem of the final state of this process with incomplete information. For solving of its problem we constructed the common algorithm that has a form of a recurrent procedure of solving a linear programming and a finite optimization problems.

INTRODUCTION

In this paper we study the problem of optimization of guaranteed result for program control by the final state of regional social and economic system in the presence of risks. For mathematical modeling of this problem we consider a discrete-time dynamical process consisting of a set a controllable objects (region and forming it municipalities). The dynamics each of these is described by the corresponding linear discrete-time recurrent vector relations and its control system consist from two levels: basic level (the level I) that is dominant level and auxiliary level (the level II) that is subordinate level. Both control levels have different criterions of functioning and united by information and control connections which defined in advance. For this problem we propose a mathematical model in the form of two-level hierarchical minimax program control problem of the final state of this process with incomplete information. For solving of its problem we constructed the common algorithm that has a form of a recurrent procedure of solving a linear programming and a finite optimization problems. Results obtained in this paper are based on the studies [1]-[5] and can be used for computer simulation, design and construction of multilevel control systems for actual economic, technical and other dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in works [1]-[3], [6], [7].

DYNAMICS OF REGION SOCIAL AND ECONOMIC CONTROL SYSTEM

On a given integer-valued time interval (simply interval) $\overline{0,T} = \{0, 1, \cdots, T\}$ $(T > 0,\ T \in \mathbb{N})$ we consider a controlled multistep dynamical process which consists of the $(n + 1)$ objects $(n \in \mathbb{N})$. Dynamics of the object I (main object of the system – region) controlled by dominant player P, is described by a vector linear discrete-time recurrent relation of the form

$$y(t + 1) = A(t)y(t) + B(t)u(t) + C(t)v(t) + D(t)w(t),\ y(0) = y_0. \tag{1}$$

The dynamics of the object II_i (ith auxiliary object of the system – ith municipality) controlled by subordinate player E_i $(i \in \overline{1,n})$, is described by the following linear relation:

$$z^{(i)}(t + 1) = A^{(i)}(t)z^{(i)}(t) + B^{(i)}(t)u(t) + C^{(i)}(t)v^{(i)}(t) + D^{(i)}(t)w^{(i)}(t),\ z^{(i)}(0) = z_0^{(i)}, \tag{2}$$

where $t \in \overline{0,T-1}$; $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbb{R}^r$ is a vector of phase variables or phase vector of the object I – a set of main parameters describing the social and economic state of a region at the time moment t; $z^{(i)}(t) =$

Application of Mathematics in Technical and Natural Sciences
AIP Conf. Proc. 1895, 050012-1–050012-11; https://doi.org/10.1063/1.5007384
Published by AIP Publishing. 978-0-7354-1579-9/$30.00

$(z_1^{(i)}(t), z_2^{(i)}(t), \ldots, z_{s_i}^{(i)}(t)) \in \mathbb{R}^{s_i}$ is a vector of phase variables or phase vector of the object II_i – a set of main parameters describing the social and economic state of the ith municipality at the time moment t; $(r, s_i \in \mathbb{N}$; for $n \in \mathbb{N}$, $\mathbb{R}^n$ is an n-dimensional Euclidean vector space of column vectors); $u(t) = (u_1(t), u_2(t), \ldots, u_p(t)) \in \mathbb{R}^p$ is a vector of control action (control) of the dominant player P at the time moment t, that satisfies the given constraint:

$$u(t) \in U_1(t) \subset \mathbb{R}^p,\ U_1(t) = \{u(t):\ u(t) = \{u^{(1)}(t), u^{(2)}(t), \cdots, u^{(N_t)}(t)\} \subset \mathbb{R}^p\}, \tag{3}$$

where $U_1(t)$ for each time moment $t \in \overline{0, T-1}$ is a finite set of vectors in the space $\mathbb{R}^p$, consisting of N_t $(N_t \in \mathbb{N})$ vectors in the space $\mathbb{R}^p$ $(p \in \mathbb{N})$; $v^{(i)}(t) = (v(i)_1(t), v(i)_2(t), \ldots, v(i)_{q_i}(t)) \in \mathbb{R}^{q_i}$ is a vector of control action (control) of the subordinate player E_i $(i \in \overline{1, n})$ at the time moment t, which depends on admissible realization of the control $u^{(j)}(t) \in \mathbf{U}_1(t)$ of the player P and must be satisfy the given constraint:

$$v^{(i)}(t) \in V_1^{(i)}(u^{(j)}(t)) \subset \mathbb{R}^{q_i},\ V_1^{(i)}(u^{(j)}(t)) = \{v^{(i)}(t):\ v^{(i)}(t) = \{v^{(i,1)}(t), v^{(i,2)}(t), \cdots, v^{(i,Q_t^{(i)}(j))}(t)\} \subset \mathbb{R}^{q_i}\}, \tag{4}$$

where $V_1^{(i)}(u^{(j)}(t))$ for each time moment $t \in \overline{0, T-1}$ and control $u^{(j)}(t) \in U_1(t)$ of the player P is the finite set of vectors in the space $\mathbb{R}^{q_i}$, consisting of $Q_t^{(i)}(j)$ $(Q_t^{(i)}(j) \in \mathbb{N})$ vectors in the space $\mathbb{R}^{q_i}$ $(q_i \in \mathbb{N})$; $v(t) = (v^{(1)}(t), v^{(2)}(t), \cdots, v^{(n)}(t)) \in \mathbb{R}^q$ is a vector of control action (control) of a subordinate player E at the time moment t, which brings together all subordinate players E_i, $i \in \overline{1, n}$ $(q = \sum_{i=1}^{n} q_i \in \mathbb{N})$.

It is assumed that for all $t \in \overline{0, T}$ each admissible realization of phase vector $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbb{R}^r$ of the object I satisfies the following given phase constraint:

$$y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in Y_1(t) \subset \mathbb{R}^r,\ Y_1(t) = \{y(t):\ y(t) \in \mathbb{R}^r,\ My(t) \leq b\} \neq \emptyset, \tag{5}$$

where the set $Y_1(t)$ constrains admissible values of realization of phase vector of the object I at the time moment t; M is real matrix of dimension $(r \times r)$; $b \in \mathbb{R}^r$ is a fixed vector; here and below in matrix inequalities symbols $\leq, =, \geq$ mean the corresponding comparison of its elements.

For all $t \in \overline{0, T}$ each admissible values of realization of phase vector $z^{(i)}(t) = (z_1^{(i)}(t), z_2^{(i)}(t), \ldots, z_{s_i}^{(i)}(t)) \in \mathbb{R}^{s_i}$ of the object II_i satisfies the following given phase constraint:

$$z^{(i)}(t) = (z_1^{(i)}(t), z_2^{(i)}(t), \ldots, z_{s_i}^{(i)}(t)) \in Z_1^{(i)}(t) \subset \mathbb{R}^{s_i},\ Z_1^{(i)}(t) = \{z^{(i)}(t):\ z^{(i)}(t) \in \mathbb{R}^{s_i},\ M^{(i)} z^{(i)}(t) \leq b^{(i)}\} \neq \emptyset, \tag{6}$$

where the set $Z_1^{(i)}(t)$ constrains admissible values of realization of phase vector of the object II_i at the time moment t; $M^{(i)}$ is real matrix of dimension $(s_i \times s_i)$; $b^{(i)} \in \mathbb{R}^{s_i}$ is a fixed vector.

In Eq. (1) describing dynamics of the object I, $w(t) = (w_1(t), w_2(t), \ldots, w_m(t))\mathbb{R}^m$ is a risks vector (or disturbances) for this object that at each time moment t $(t \in \overline{0, T-1})$ depends on admissible realization of the control $u^{(j)}(t) \in U_1(t)$ of the player P $(j \in \overline{1, N_t})$ and satisfies the given constraint:

$$w(t) = (w_1(t), w_2(t), \ldots, w_m(t)) \in W_1(u^{(j)}(t)) \subset \mathbb{R}^m,\ W_1(u^{(j)}(t)) = \{w(t):\ w(t) \in \mathbb{R}^m,\ Rw(t) + Lu^{(j)}(t) \leq c\} \neq \emptyset, \tag{7}$$

i.e., the set $W_1(u^{(j)}(t))$ constrains possible values of realization of risks vectors $w(t)$ at the time moment t which influences dynamics of the object I. In the constraint (7): R and L are real matrices of dimensions $(m \times r)$ and $(m \times p)$ respectively; $c \in \mathbb{R}^m$ is a fixed vector.

In the equation (2) describing dynamics of the object II_i $(i \in \overline{1, n})$, $w^{(i)}(t) = (w_1^{(i)}(t), w_2^{(i)}(t), \ldots, w_{m_i}^{(i)}(t)) \in \mathbb{R}^{m_i}$ is a risks vector (or disturbances) for this object that at each time moment t $(t \in \overline{0, T-1})$ depends on admissible realization of the control $u^{(j)}(t) \in U_1(t)$ of the player P $(j \in \overline{1, N_t})$ and satisfies the given constraint:

$$w^{(i)}(t) = (w_1^{(i)}(t), w_2^{(i)}(t), \ldots, w_{m_i}^{(i)}(t)) \in W_1^{(i)}(u^{(j)}(t)) \subset \mathbb{R}^{m_i},$$

$$W_1^{(i)}(u^{(j)}(t)) = \{w^{(i)}(t):\ w^{(i)}(t) \in \mathbb{R}^{m_i},\ R^{(i)} w^{(i)}(t) + L^{(i)} u^{(j)}(t) \leq c^{(i)}\} \neq \emptyset, \tag{8}$$

i.e., the set $W_1^{(i)}(u^{(j)}(t))$ constrains possible values of realization of risks vectors $w^{(i)}(t)$ at the time moment t, which influences dynamics of the object II_i. In the constraint (8): $R^{(i)}$ and $L^{(i)}$ are real matrices of dimensions $(m_i \times m_i)$ and $(m_i \times p)$ respectively; $c^{(i)} \in \mathbb{R}^{m_i}$ is a fixed vector.

Matrices $A(t)$, $B(t)$, $C(t)$, and $D(t)$ in the vector recurrent equation (1), describing dynamics of the object I, are real matrices of dimensions $(r \times r)$, $(r \times p)$, $(r \times q)$, and $(r \times m)$ respectively. In vector recurrent equation (2), describing dynamics of the object II_i $(i \in \overline{1, n})$, matrices $A^{(i)}(t)$, $B^{(i)}(t)$, $C^{(i)}(t)$, and $D^{(i)}(t)$ are real matrices of dimensions $(s_i \times s_i)$, $(s_i \times p_i)$, $(s_i \times q_i)$, and $(s_i \times m_i)$ respectively.

INFORMATION CONDITIONS FOR THE PLAYERS IN THE CONTROL PROCESS

The control system for the discrete-time dynamical process (1)-(8) are realized in the presence of the following information conditions.

It is assumed that in the field of interests of the player P are all possible terminal (final) states $y(T)$ of the object I and possible states $z^{(i)}(T)$ of all objects $II_i,\ i \in \overline{1,n}$. And for any considered time interval $\overline{t,T} \subseteq \overline{0,T}$ $(t < T)$ the player P also knows a future realization of the program control $v^{(i)}(\cdot) = \{v^{(i)}(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ v^{(i)}(\tau) \in V_1^{(i)}(u^{(j)}(\tau)),$ $u^{(j)}(\tau) \in U_1(\tau), j \in N_\tau)$ of the each player E_i $(i \in \overline{1,n})$ at this time interval which communicate to him, and he can use its for constructing his program control $u(\cdot) = \{u(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ u(\tau) \in U_1(\tau))$.

We assumed that in the field of interests of each player E_i $(i \in \overline{1,n})$ are only possible terminal states $z^{(i)}(T)$ of the object II_i and for any considered time interval $\overline{t,T} \subseteq \overline{0,T}$ $(t < T)$ he also knows a future realization of the program control $u(\cdot) = \{u(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ u(\tau) \in U_1(\tau))$ of the player P at this time interval, which communicate to him, and he can use its for constructing his program control $v^{(i)}(\cdot) = \{v^{(i)}(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ v^{(i)}(\tau) \in V_1^{(i)}(u^{(j)}(\tau)),\ u^{(j)}(\tau) \in U_1(\tau), j \in N_\tau)$.

We also assumed that in the considered control system for every time moment $t \in \overline{0,T}$ players P and $E_i,\ i \in \overline{1,n}$, knows all relations and constraints describing the the dynamical process (1)-(8).

Then on base of the given assumptions we will say that such possibilities of the behavior of player P combined with the players E_i, and objects I and $II_i,\ i \in \overline{1,n}$, are defined as the level I or the dominant level of the control system for considered dynamical process (1)-(8).

The collection n of players $E_i,\ i \in \overline{1,n}$ which will be called as player E and objects $II_i,\ i \in \overline{1,n}$ controlled by them form the level II or the subordinate level of the control system for considered dynamical process (1)-(8) (which is subordinate to the level I or the dominant level of the control system).

It is assumed that the player P estimate the result of the realization of the dynamical process (1)-(8) by the values of the linear functional $\hat{\alpha}$, which is defined on the final phase states $y(T)$ and $z^{(i)}(T)$ of the objects I and II_i, $i \in \overline{1,n}$, respectively. Note that this functional estimate the state of social and economic parameters of the region and its constituent municipalities for the player P on the level I of the control system. Each player II_i $(i \in \overline{1,n})$ estimate the result of the realization of this dynamical process (1)-(8) by the values of the linear functional $\hat{\beta}^{(i)}$, which is defined only on the final phase states $z^{(i)}(T)$ of the object II_i. This functional estimate the state of social and economic parameters of the ith municipality for the player E_i on the level II of the control system.

The aim of player P on the level I of this control system and fixed time interval $\overline{t,T} \subseteq \overline{0,T}$ $(t < T)$ can be formulate in the following way. The player P using his information and control possibilities has interest in such result of the realization of the dynamical process (1)-(8) on the interval $\overline{t,T}$ when functional $\hat{\alpha}$ has minimal admissible value at worst for him realization of risk vectors $w(\cdot) = \tau_{\tau \in \overline{t,T-1}}$ and $w^{(i)}(\cdot) = \tau_{\tau \in \overline{t,T-1}}$. And this aim he can realize by the way a choice of his program control $u(\cdot) = \{u(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ u(\tau) \in U_1(\tau))$ and on the base of program control $v^{(i)}(\cdot) = \{v^{(i)}(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ v^{(i)}(\tau) \in V_1^{(i)}(u^{(j)}(\tau)),\ u^{(j)}(\tau) \in U_1(\tau), j \in N_\tau)$ of the players $E_i,\ i \in \overline{1,n}$, at this time interval, which communicate to him. Note, that each player E_i to help him in achieving its aim.

Then the aim of each player E_i $(i \in \overline{1,n})$ on the level II of the control system and fixed interval $\overline{t,T} \subseteq \overline{0,T}$ $(t < T)$ can be formulate in the following way. The player E_i $(i \in \overline{1,n})$ using his information and control possibilities has interest in such result of the realization of the dynamical process (1)-(8) on the interval $\overline{t,T}$ when functional $\hat{\beta}^{(i)}$ has minimal admissible value at worst for him realization of risk vector $w^{(i)}(\cdot) = \tau_{\tau \in \overline{t,T-1}}$. And this aim he can realize by the way a choice his program control $v^{(i)}(\cdot) = \{v^{(i)}(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ v^{(i)}(\tau) \in V_1^{(i)}(u^{(j)}(\tau)),\ u^{(j)}(\tau) \in U_1(\tau), j \in N_\tau)$ on the base of program control $u(\cdot) = \{u(\tau)\}_{\tau \in \overline{t,T-1}}$ $(\forall\ \tau \in \overline{t,T-1}:\ u(\tau) \in U_1(\tau))$ of the player P at this time interval, which communicate to him.

DEFINITIONS AND CRITERIONS OF QUALITY FOR THE CONTROL PROCESS

For a strict mathematical formulation the two-level hierarchical minimax program control problem by a final states phase vectors in discrete-time dynamical process (1)-(8) with risks we introduce some definitions.

For a fixed number $k \in \mathbb{N}$ and an integer-valued interval $\overline{\tau,\vartheta} \subseteq \overline{0,T}$ $(\tau \leq \vartheta)$, we denote by $\mathbb{S}_k(\overline{\tau,\vartheta})$ the metric space of functions $\varphi : \overline{\tau, \vartheta} \longrightarrow \mathbb{R}^k$ of an integer argument t where the metric ρ_k is defined as

$$\rho_k(\varphi_1(\cdot), \varphi_2(\cdot)) = \max_{t \in \overline{\tau, \vartheta}} \| \varphi_1(t) - \varphi_2(t) \|_k \ \ ((\varphi_1(\cdot), \varphi_2(\cdot)) \in \mathbb{S}_k(\overline{\tau, \vartheta}) \times \mathbb{S}_k(\overline{\tau, \vartheta}));$$

by $\mathrm{comp}(\mathbb{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact (in the sense of this metric) subsets of the space $\mathbb{S}_k(\overline{\tau, \vartheta})$. Here for $x \in \mathbb{R}^k$ in what follows $\| x \|_k$ denotes the Euclidean norm of vector x in the space $\mathbb{R}^k$.

Based on constraint (3) we define the set $\mathbf{U}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbb{S}_p(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ of the player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ by the relation

$$\mathbf{U}(\overline{\tau, \vartheta}) = \{u(\cdot) : \ u(\cdot) \in \mathbb{S}_p(\overline{\tau, \vartheta - 1}), \forall \ t \in \overline{\tau, \vartheta - 1}, \ u(t) \in U_1(t)\}.$$

Similarly, for a fixed program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P and index $i \in \overline{1, n}$, according to constraint (4) we define the set $\mathbf{V}^{(i)}(\overline{\tau, \vartheta}; u(\cdot))$ of all admissible program controls of the player E_i on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ of the corresponding $u(\cdot)$, and according to constraints (7) and (8) we define the sets $\mathbf{W}(\overline{\tau, \vartheta}; u(\cdot))$ and $\mathbf{W}^{(i)}(\overline{\tau, \vartheta}; u(\cdot))$ $i \in \overline{1, n}$ respectively of all admissible program risk vector of modeling dynamics of the objects I and II_i on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ and corresponding the control $u(\cdot)$.

For a fixed program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P we also introduce the sets: $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot)) = \prod_{i=1}^{n} \mathbf{V}^{(i)}(\overline{\tau, \vartheta}; u(\cdot)$, $\hat{\mathbf{W}}(\overline{\tau, \vartheta}; u(\cdot)) = \prod_{i=1}^{n} \mathbf{W}^{(i)}(\overline{\tau, \vartheta}; u(\cdot))$, which are the sets of all admissible collections $v(\cdot) = \{v^{(1)}(\cdot), v^{(2)}(\cdot), \cdots, v^{(n)}(\cdot)\} \in \prod_{i=1}^{n} \mathbf{V}^{(i)}(\overline{\tau, \vartheta})$ of program controls for company players $E_i, i \in \overline{1, n}$, or all admissible program controls $v(\cdot)$ of the player E, and all admissible collections $\hat{w}(\cdot) = \{w^{(1)}(\cdot), w^{(2)}(\cdot), \cdots, w^{(n)}(\cdot)\} \in \prod_{i=1}^{n} \mathbf{W}^{(i)}(\overline{\tau, \vartheta}; u(\cdot))$ of risk vectors of modeling dynamics of objects II_i, $i \in \overline{1, n}$, respectively, and each of them defined on the interval $\overline{\tau, \vartheta}$.

Let for time moment $\tau \in \overline{0, T}$ the set $\mathbf{G}(\tau) = \overline{0, T} \times \mathbb{R}^r \times \prod_{i=1}^{n} \mathbb{R}^{s_i}$ be the set of all admissible τ-positions $g(\tau) = \{0, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \overline{0, T} \times \mathbb{R}^r \times \prod_{i=1}^{n} \mathbb{R}^{s_i}$ of the player P ($\mathbf{G}(0) = \{g(0)\} = \mathbf{G}_0 = \{g_0\}$, $g(0) = g_0 = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\}$) satisfying the constrains (5) and (6).

Then, for any interval $\overline{\tau, T} \subset \overline{0, T}$, and admissible realizations of τ-position $g(\tau) \in \mathbf{G}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$, and program risk vectors $w(\cdot) \in \mathbf{W}(\overline{\tau, T})$ and $\hat{w}(\cdot) \in \hat{\mathbf{W}}(\overline{\tau, T}; u(\cdot))$ for estimating by the player P on the level I of the control system the quality of realization of final phase states of the object I and objects II_i, $i \in \overline{1, n}$, we define the following terminal functional

$$\alpha : \ \mathbf{G}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \mathbf{V}(\overline{\tau, T}) \times \mathbf{W}(\overline{\tau, T}) \times \hat{\mathbf{W}}(\overline{\tau, T}) = \mathbf{\Omega}(\overline{\tau, T}, \alpha) \longrightarrow \mathbb{E} =]-\infty, +\infty[, \tag{9}$$

and its values are determined by the following concrete relation

$$\alpha(g(\tau), u(\cdot), v(\cdot), w(\cdot), \hat{w}(\cdot)) = \hat{\alpha}(y(T), z^{(1)}(T), z^{(2)}(T), \cdots, z^{(n)}(T))$$

$$= \mu \cdot \hat{\gamma}(y(T)) + \sum_{i=1}^{n} \mu^{(i)} \cdot \hat{\beta}^{(i)}(z^{(i)}(T)) = \mu \cdot < e, y(T) >_r + \sum_{i=1}^{n} \mu^{(i)} \cdot < e^{(i)}, z^{(i)}(T) >_{s_i} . \tag{10}$$

Where by $y(T) = y_T(\overline{\tau, T}, y(\tau), u(\cdot), v(\cdot), w(\cdot))$ and by $z^{(i)}(T) = z_T^{(i)}(\overline{\tau, T}, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), w^{(i)}(\cdot))$ we denote the sections of motions of object I and object II_i $(i \in \overline{1, n})$ respectively, at final instant T on the interval $\overline{\tau, T}$; $\mu \in \mathbb{R}^1$ and $\mu^{(i)} \in \mathbb{R}^1$ $(i \in \overline{1, n})$ are concrete numerical parameters which satisfying the following conditions:

$$\mu \geq 0; \ \forall \ i \in \overline{1, n}, \ \mu^{(i)} \geq 0; \ \sum_{i=1}^{n} \mu^{(i)} = 1 - \mu; \tag{11}$$

$\hat{\alpha}:\ \mathbb{R}^r \times \mathbb{R}^s \to \mathbb{R}^1$ is linear terminal functional ($\sum_{i=1}^{n} s_i = s$); $\hat{\gamma}:\ \mathbb{R}^r \to \mathbb{R}^1$ is linear terminal functional; $\hat{\beta}^{(i)}:\ \mathbb{R}^{s_i} \to \mathbb{R}^1$ is linear terminal functional; $e \in \mathbb{R}^r$ and $e^{(i)} \in \mathbb{R}^{s_i},\ i \in \overline{1,n}$, are fixed vectors; here and below, for each $k \in \mathbb{N},\ a \in \mathbb{R}^k$ and $b \in \mathbb{R}^k$ will be denoted by the symbol $< a, b >_k$ scalar product of vectors a and b of the space $\mathbb{R}^k$.

We denote by $\mathbf{G}^{(i)}(\tau) = \overline{0,T} \times \mathbb{R}^{s_i}$ the set of all admissible τ-positions $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \overline{0,T} \times \mathbb{R}^{s_i}$ of the player E_i ($i \in \overline{1,n}$; $\mathbf{G}^{(i)}(0) = \{g^{(i)}(0)\} = \hat{\mathbf{G}}_0^{(i)} = \{g_0^{(i)}\},\ g^{(i)}(0) = g_0^{(i)} = \{0, z_0^{(i)}\}$), and by $\hat{\mathbf{G}}(\tau) = \overline{0,T} \times \prod_{i=1}^{n} \mathbb{R}^{s_i}$ ($\hat{\mathbf{G}}(0) = \{\hat{g}(0)\} = \hat{\mathbf{G}}_0 = \{\hat{g}_0\},\ \hat{g}(0) = \hat{g}_0 = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\}$) the set of all admissible τ-positions $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \overline{0,T} \times \prod_{i=1}^{n} \mathbb{R}^{s_i}$ for all company of the players $E_i,\ i \in \overline{1,n}$, or the player E, for level II of the control process satisfying the constrain (6).

Then, for any interval $\overline{\tau,T} \subset \overline{0,T}$, and admissible realizations of τ-position $g^{(i)}(\tau) \in \mathbf{G}^{(i)}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ and $v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot))$, and program risk vector $w^{(i)}(\cdot) \in \hat{\mathbf{W}}^{(i)}(\overline{\tau,T}; u(\cdot))$, for estimating by each player E_i ($i \in \overline{1,n}$) on the level II of the control system the quality of realization of final phase states of the object II_i we define the following terminal functional, namely

$$\beta^{(i)}:\ \mathbf{G}^{(i)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot)) \times \mathbf{W}^{(i)}(\overline{\tau,T}; u(\cdot)) = \mathbf{\Omega}(\overline{\tau,T}, \beta^{(i)}) \longrightarrow \mathbb{E}, \tag{12}$$

and its values are defined by the following concrete relation

$$\beta^{(i)}(g^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), w^{(i)}(\cdot)) = \hat{\beta}^{(i)}(z^{(i)}(T)) =< e^{(i)}, z^{(i)}(T) >_{s_i}, \tag{13}$$

where the linear terminal functional $\hat{\beta}^{(i)}$ is from the relation (10).

OPTIMIZATION PROBLEMS FOR THE CONTROL PROCESS

On the basis of the assumptions made above for realization the aim of the player E_i ($i \in \overline{1,n}$) we can formulate the following minimax program control problem by a final state phase vector of the object II_i on the level II of the control system for dynamical process (1)-(8).

Problem 1. For fixed index $i \in \overline{1,n}$, interval $\overline{\tau,T} \subseteq \overline{0,T}$ ($\tau < T$), admissible on the level II in the two level hierarchical control system for the dynamical process (1)-(8) realization τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \mathbf{G}^{(i)}(\tau)$ ($g^{(i)}(0) = g_0^{(i)} \in \mathbf{G}_0^{(i)}$) of the player E_i, and every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T}$ of the player P on the level I of this control system it is required to find the set $\mathbf{V}^{(i,e)}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot)) \subseteq \mathbf{V}^{(i)}(\overline{\tau,T}; (u(\cdot))$ of minimax program controls $v^{(i,e)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot))$ of the player E_i on the level II of the control system for the dynamical process (1)-(8) corresponding the control $u(\cdot)$ of the player P and it set is defined by the following relation

$$\begin{aligned}
\mathbf{V}^{(i,e)}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot)) = \{v^{(i,e)}(\cdot):\ v^{(i,e)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot)),\ c^{(e)}_{\beta^{(i)}}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot)) \\
= \max_{w^{(i)}(\cdot) \in \mathbf{W}^{(i)}(\overline{\tau,T}; u(\cdot))} \beta^{(i)}(g^{(i)}(\tau), v^{(i,e)}(\cdot), u(\cdot), w(i)(\cdot)) \\
= \min_{v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot))} \max_{w^{(i)}(\cdot) \in \mathbf{W}^{(i)}(\overline{\tau,T}; u(\cdot))} \beta^{(i)}(g^{(i)}(\tau), v^{(i)}(\cdot), u(\cdot), w^{(i)}(\cdot))\},
\end{aligned} \tag{14}$$

where functional $\beta^{(i)}$ is defined by the relations (12) and (13).

We call the value of the number $c^{(e)}_{\beta^{(i)}}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot))$ as the value of the result of the minimax program control of the player E_i on the level II of the control system, and we call the set $\mathbf{V}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u(\cdot)) = \prod_{i=1}^{n} \mathbf{V}^{(i,e)}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot))$, which formed due from solving of the n Problems 1 for all $i \in \overline{1,n}$, as the set of minimax program controls of the player E on the level II of the control system for the dynamical process (1)-(8) and corresponding to it the value of the vector $c^{(e)}_{\beta}(\overline{\tau,T}, \hat{g}(\tau), u(\cdot)) = (c^{(e)}_{\beta^{(1)}}(\overline{\tau,T}, g^{(1)}(\tau), u(\cdot)), c^{(e)}_{\beta^{(2)}}(\overline{\tau,T}, g^{(2)}(\tau), u(\cdot)), \cdots, c^{(e)}_{\beta^{(n)}}(\overline{\tau,T}, g^{(n)}(\tau), u(\cdot))) \in \mathbb{E}^n$ we call as the value of the result of the minimax program control of the player E on the level II of the control system (where τ-position $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau) = \prod_{i=1}^{n} \mathbf{G}^{(i)}(\tau)$ ($\hat{g}(0) = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = \hat{g}_0 \in \hat{\mathbf{G}}_0$). It should be noted that the vector $c^{(e)}_{\beta}(\overline{\tau,T}, g(\tau), u(\cdot))$ is define the concrete value of the vector functional $\beta = (\beta^{(1)}, \beta^{(2)}, \cdots, \beta^{(n)})$ which defined by the relation (13) and such that may be determine by the following mapping $\beta : \prod_{i=1}^{n} \mathbf{\Omega}(\overline{\tau,\mathrm{T}}, \beta^{(i)}) \longrightarrow \mathbb{E}^n$, where for each index $i \in \overline{1,n}$ the value of the functional $\beta^{(i)}$ is defined by formula (13).

Note, that we can use the vector functional β as quality test of behavior of the player E (or company of all players $E_i,\ i \in \overline{1,n}$) on the level II of this control system in situation when all players $E_i,\ i \in \overline{1,n}$ have common aim and they organize common coalition.

Below, for realization the aim of the player P corresponding by the level I of considered two-level hierarchical control system we formulate the following minimax program control problem by a final state phase vectors of the objects I and $II_i,\ i \in \overline{1,n}$ in the dynamical process (1)-(8).

Problem 2. For fixed time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible on the level I of the two-level hierarchical control system for the dynamical process (1)-(8) realization τ-position $g(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \mathbf{G}(\tau)$ $(g(0) = \{0, y_0, z^{(1)}_0, z^{(2)}_0, \cdots, z^{(n)}_0\} = g_0 \in \mathbf{G}_0)$ of the player P it is required to find the set $\mathbf{U}^{(e)}(\overline{\tau,T}, g(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$ of the minimax program controls of the player P which defined by the following relation

$$\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau)) = \{u^{(e)}(\cdot) :\ u^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,T}),\ c^{(e)}_{\alpha}(\overline{\tau,T}, g(\tau))$$

$$= \min_{v^{(e)}(\cdot)\in\mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))} \ \max_{\substack{w(\cdot)\in\mathbf{W}(\overline{\tau,T};u^{(e)}(\cdot))\\ \hat{w}(\cdot)\in\hat{\mathbf{W}}(\overline{\tau,T};u^{(e)}(\cdot))}} \{\alpha(g(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), w(\cdot), \hat{w}(\cdot))\}$$

$$= \min_{u(\cdot)\in\mathbf{U}(\overline{\tau,T})} \ \min_{v^{(e)}(\cdot)\in\mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot))} \ \max_{\substack{w(\cdot)\in\mathbf{W}(\overline{\tau,T})\\ \hat{w}(\cdot)\in\hat{\mathbf{W}}(\overline{\tau,T})}} \alpha(g(\tau), u(\cdot), v^{(e)}(\cdot), w(\cdot), \hat{w}(\cdot))\}. \tag{15}$$

Where the functional α defined by relations (9) and (10); τ-position $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(\hat{g}(0) = \{0, z^{(1)}_0, z^{(2)}_0, \cdots, z^{(n)}_0\} = \hat{g}_0 \in \hat{\mathbf{G}}_0)$ of the player E formed due from τ-position $g(\tau)$ of the player P and determines the realization at instant τ the phase states all the objects $II_i,\ i \in \overline{1,n}$ on the level II of this control system for the dynamical process (1)-(8) and the set $\mathbf{V}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u(\cdot)) = \{v^{(e)}(\cdot) = \{v^{(1,e)}(\cdot), v^{(2,e)}(\cdot), \cdots, v^{(n,e)}(\cdot)\}\} \subseteq \mathbf{V}(\overline{\tau,T}; u(\cdot))$ of minimax program controls of the player E for level II of the control system for any realizations τ-position $\hat{g}(\tau) \in \hat{\mathbf{G}}(\tau)$ $(\hat{g}(0) = \hat{g}_0 \in \hat{\mathbf{G}}_0)$ of the player E and program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of the player P which formed from solving of the Problems 1 for all values of the parameter $i \in \overline{1,n}$.

The set $\mathbf{U}^{(e)}(\overline{\tau,T}, g(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$ which is forming from solving of the Problems 1 and 2 we call as the set of minimax program controls of the player P on the level I of the two-level hierarchial control system for the dynamical process (1)-(8) and corresponding to it the number $c^{(e)}_{\alpha}(\overline{\tau,T}, g(\tau))$ we call as the value of the result of the program minimax control for the player P on the level I of this control system.

On the base of formulated the Problems 1 and 2 we consider the following problem.

Problem 3. For fixed time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible on the level I of two-level hierarchical program control system for the dynamical process (1)-(8) realization the τ-position $g(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \mathbf{G}(\tau)$ $(g(0) = \{0, y_0, z^{(1)}_0, z^{(2)}_0, \cdots, z^{(n)}_0\} = g_0 \in \hat{\mathbf{W}}_0)$ of the player P and admissible on the level II of the control system the realization τ-position $\hat{g}(\tau) \in \hat{\mathbf{G}}(\tau)$ $(\hat{g}(0) = \hat{g}_0 \in \hat{\mathbf{G}}_0)$ of the player E, which formed due from the τ-position $g(\tau)$, and admissible realization of the program minimax control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,T}, g(\tau))$ of the player P on the level I of the control system, which formed from solving of the Problems 1 and 2, it is required to find the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau,T}; u^{(e)}(\cdot))$ of the optimal minimax program controls $\hat{v}^{(e)}(\cdot) = \{\hat{v}^{(1,e)}(\cdot), \hat{v}^{(2,e)}(\cdot), \cdots, \hat{v}^{(n,e)}(\cdot)\} \in \mathbf{V}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u^{(e)}(\cdot))$ of the player E on the level II of the control system and vector $c^{(e)}_{\beta}(\overline{\tau,T}, \hat{g}(\tau), u^{(e)}(\cdot)) = (c^{(e)}_{\beta^{(1)}}(\overline{\tau,T}, g^{(1)}(\tau), u^{(e)}(\cdot)), c^{(e)}_{\beta^{(2)}}(\overline{\tau,T}, g^{(2)}(\tau), u^{(e)}(\cdot)), \cdots, c^{(e)}_{\beta^{(n)}}(\overline{\tau,T}, g^{(n)}(\tau), u^{(e)}(\cdot))) \in \mathbb{E}^n$ of optimal value of the result of the minimax program control for the player E on the level II of the control system for considered dynamical process and corresponding to the control $u^{(e)}(\cdot)$ of the player P and defined by the following relations:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u^{(e)}(\cdot)) = \{\hat{v}^{(e)}(\cdot) :\ \hat{v}^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau,T}, \hat{g}(\tau), u^{(e)}(\cdot)),\ c^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau))$$

$$= \max_{\substack{w(\cdot)\in\mathbf{W}(\overline{\tau,T};u^{(e)}(\cdot))\\ \hat{w}(\cdot)\in\hat{\mathbf{W}}(\overline{\tau,T};u^{(e)}(\cdot))}} \alpha(g(\tau), u^{(e)}(\cdot), \hat{v}^{(e)}(\cdot), w(\cdot), \hat{w}(\cdot)) = \min_{v^{(e)}(\cdot)\in\mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))} \ \max_{\substack{w(\cdot)\in\mathbf{W}(\overline{\tau,T};u^{(e)}(\cdot))\\ \hat{w}(\cdot)\in\hat{\mathbf{W}}(\overline{\tau,T};u^{(e)}(\cdot))}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), w(\cdot), \hat{w}(\cdot))\}; \tag{16}$$

$$\forall\, i \in \overline{1,n}:\ c^{(e)}_{\beta^{(i)}}(\overline{\tau,T}, g^{(i)}(\tau), u^{(e)}(\cdot)) = \max_{w^{(i)}(\cdot)\in \mathbf{W}^{(i)}(\overline{\tau,T};u^{(e)}(\cdot))} \beta^{(i)}(g^{(i)}(\tau), \hat{v}^{(i,e)}(\cdot), u^{(e)}(\cdot), w^{(i)}(\cdot))$$

$$= \min_{v^{(i)}(\cdot)\in \mathbf{V}^{(i)}(\overline{\tau,T};u^{(e)}(\cdot))} \max_{w^{(i)}(\cdot)\in \mathbf{W}^{(i)}(\overline{\tau,T};u^{(e)}(\cdot))} \beta^{(i)}(g^{(i)}(\tau), v^{(i)}(\cdot), u^{(e)}(\cdot), w^{(i)}(\cdot)). \qquad (17)$$

Note, that we can consider the solutions of formulated Problems 1-3 which in union are determined the solution of the main problem of two-level hierarchical minimax program control by the final state of regional social and economic system in the presence of risks for the dynamical process (1)-(8).

ALGORITHMS OF SOLVING THE PROBLEMS 1-3

Thus, for any fixed and admissible time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and realization τ-position $g(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \mathbf{G}(\tau)$ $(g(0) = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = g_0 \in \mathbf{G}_0)$ of the player P on the level I of the two-level hierarchical control system for the dynamical process (1)-(8) and corresponding to it τ-position $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(g(0) = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = g_0 \in \hat{\mathbf{G}}_0)$ of the player E on the level II of this control system we can describe the algorithm for solving Problems 1-3 formulated above.

For fixed collection $(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot)) \in \{\tau\} \times \mathbb{R}^{s_i} \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot))$ $(i \in \overline{1,n})$ we define by virtue of (2), (4), and (8) the following set:

$$\mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T) = \{z^{(i)}(T):\ z^{(i)}(T) \in \mathbb{R}^{s_i},\ \forall\, t \in \overline{\tau, T-1},\ z^{(i)}(t+1)$$

$$= A^{(i)}(t)z^{(i)}(t) + B^{(i)}(t)u(t) + C^{(i)}(t)v^{(i)}(t) + D^{(i)}(t)w^{(i)}(t) \in Z_1^{(i)}(t+1),\ (z^{(i)}(\tau), w^{(i)}(\cdot)) \in \{z^{(i)}(\tau)\} \times \mathbf{W}^{(i)}(\overline{\tau,T}; u(\cdot))\}, \quad (18)$$

where $\mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T)$ is a reachable set [1], [2] of all admissible phase states of the object II_i at time moment T corresponding to the collection $(z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot))$, and satisfying the constrain (6).

We fix time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$, admissible on the level II of the control system realization τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \mathbf{G}^{(i)}(\tau)$ $(g^{(i)}(0) = g_0^{(i)} \in \mathbf{G}_0^{(i)})$ of the player E_i and any admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system.

Then, on the basis of the above definitions and results of the works [3], [4] the procedure of construction solution of the Problem 1 for dynamical process (1)-(8) can be represented as a sequence consisting from solving of the following four sub-problems:

1) constructing for every admissible control $v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot))$ of the player E_i of the reachable set $\mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T)$ (note, that this set can be constructed by finding a solutions of a finite sequence a linear mathematical programming problems, and this set is convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbb{R}^{s_i}$ [3], [4], [5]);

2) maximizing of the linear terminal functional $\hat{\beta}^{(i)}$ which is defined by the relation (13) on the set $\mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T)$, namely, to construct the following number:

$$\kappa^{(i,e)}_{\beta^{(i)}}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot)) = < e^{(i)}, \tilde{z}^{(i,e)}(T) >_{s_i} = \max_{z^{(i)}(T)\in \mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T)} < e^{(i)}, z^{(i)}(T) >_{s_i} \qquad (19)$$

(note, that the solution of this problem is reduced to solving a linear mathematical programming problem [3], [4]);

3) constructing of the set $\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T}, w^{(i)}(\tau), u(\cdot))$ and the number $\tilde{c}^{(i,e)}_{\beta}(\overline{\tau,T}, w^{(i)}(\tau), u(\cdot))$ from solving the following optimization problem:

$$\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T}, w^{(i)}(\tau), u(\cdot)) = \{\tilde{v}^{(i,e)}(\cdot):\ \ \tilde{v}^{(i,e)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot)),\ \tilde{c}^{(i,e)}_{\beta^{(i)}}(\overline{\tau,T}, w^{(i)}(\tau), u(\cdot))$$

$$= \kappa^{(i,e)}_{\beta^{(i)}}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot), \tilde{v}^{(i,e)}(\cdot)) = \min_{v^{(i)}(\cdot)\in \mathbf{V}^{(i)}(\overline{\tau,T};u(\cdot))} \kappa^{(i,e)}_{\beta^{(i)}}(\overline{\tau,T}, g^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot))\} \qquad (20)$$

(note, that the set $\mathbf{V}^{(i)}(\overline{\tau,T}; u(\cdot))$ is a finite set at the space $\mathbb{R}^{q_i}$, and then the solution of this problem is reduced to solving the finite discrete optimization problem);

4) constructing of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot)) = \prod_{i=1}^{n}\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$ and the number $\tilde{c}_{\beta}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot)) = (\tilde{c}_{\beta^{(1)}}^{(e)}(\overline{\tau,T},g^{(1)}(\tau),u(\cdot)),\tilde{c}_{\beta^{(2)}}^{(e)}(\overline{\tau,T},g^{(2)}(\tau),u(\cdot)),\cdots,\tilde{c}_{\beta^{(n)}}^{(e)}(\overline{\tau,T},g^{(n)}(\tau),u(\cdot))) \in \mathbb{E}^n$ which formed due from solving of the n problems describing by the relation (20) for each $i \in \overline{1,n}$.

Taking into consideration (12), (13), (14), (18)-(20), and the conditions stipulated for the system (1)-(8), one can prove (analogy as in works [3], [4]), that the following assertion is valid.

Theorem 1. For fixed index $i \in \overline{1,n}$, time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$, admissible on the level II of the two level hierarchical control system for the discrete-time dynamical process (1)-(8) realization τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \mathbf{G}^{(i)}(\tau)$ of the player E_i $(i \in \overline{1,n})$, and for every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system, the set $\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$ of the admissible program controls $\tilde{v}^{(i,e)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,T};u(\cdot))$ of the player E_i on the level II of the control system and the number $\tilde{c}_{\beta}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$ constructed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problem, and the following equalities are true:

$$\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot)) = \mathbf{V}^{(i,e)}(\overline{\tau,T},w^{(1)}(\tau),u(\cdot));\ \tilde{c}_{\beta^{(i)}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot)) = c_{\beta^{(i)}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot)), \tag{21}$$

where the set $\mathbf{V}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$ and the number $c_{\beta^{(i)}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$ satisfies the relation (14).

Then from this assertion follows that a solution of the Problem 1 for the discrete-time dynamical process (1)-(8) can be formed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problem on the basis of construction of the set $\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$ and the number $\tilde{c}_{\beta^{(i)}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot))$, and the following equalities are true:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot)) = \prod_{i=1}^{n}\tilde{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot)) = \prod_{i=1}^{n}\mathbf{V}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),u(\cdot)) = \mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot)),$$

where $\mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot))$ is the set of minimax program controls of the player E on the level II of the two level hierarchical control system for the dynamical process (1)-(8);

$$\tilde{c}_{\beta}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot)) = (\tilde{c}_{\beta^{(1)}}^{(e)}(\overline{\tau,T},g^{(1)}(\tau),u(\cdot)),\tilde{c}_{\beta^{(2)}}^{(e)}(\overline{\tau,T},g^{(2)}(\tau),u(\cdot)),\cdots,\tilde{c}_{\beta^{(n)}}^{(e)}(\overline{\tau,T},g^{(n)}(\tau),u(\cdot)))$$

$$= (c_{\beta^{(1)}}^{(e)}(\overline{\tau,T},g^{(1)}(\tau),u(\cdot)),c_{\beta^{(2)}}^{(e)}(\overline{\tau,T},g^{(2)}(\tau),u(\cdot)),\cdots,c_{\beta^{(n)}}^{(e)}(\overline{\tau,T},g^{(n)}(\tau),u(\cdot))) = c_{\beta}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot)) \in \mathbb{E}^n,$$

where $c_{\beta}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u(\cdot))$ is the value of the result of the minimax program control of the player E on the level II of the control system for this dynamical process; τ-position $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau) = \prod_{i=1}^{n}\mathbf{G}^{(i)}(\tau)$ $(\hat{g}(0) = \{0, z_0^{(1)}, z_0^{(2)},\cdots,z_0^{(n)}\} = \hat{g}_0 \in \hat{\mathbf{G}}_0)$.

Next, consider the algorithm for solving of the Problem 2.

For fixed collection $(\tau, y(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times Y_1(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}(\overline{\tau,T};u(\cdot))$ we define by virtue of (1), (3), and (5) the following set:

$$\mathbf{Y}(\tau,y(\tau),u(\cdot),v(\cdot),Y_1(\cdot),T) = \{y(T):\ y(T) \in \mathbb{R}^r,\ \forall\, t \in \overline{\tau,T-1},$$

$$y(t+1) = A(t)y(t) + B(t)u(t) + C(t)v(t) + D(t)w(t) \in Y_1(t+1),\ (y(\tau),w(\cdot)) \in \{y(\tau)\} \times \mathbf{W}(\overline{\tau,T};u(\cdot))\}, \tag{22}$$

where $\mathbf{Y}(\tau,y(\tau),u(\cdot),v(\cdot),Y_1(\cdot),T)$ is a reachable set [1], [2] of all admissible phase states of the object I at time moment T corresponding to the collection $(\tau,y(\tau),u(\cdot),v(\cdot),Y_1(\cdot))$, and satisfying the constrain (5).

We fix time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$, admissible on the level I and II of the two level hierarchical control system for the dynamical process (1)-(8) realizations τ-positions $g(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\} \in \mathbf{G}(\tau)$ $(g(0) = \{0, y_0, z_0^{(1)}, z_0^{(2)},\cdots,z_0^{(n)}\} = g_0 \in \mathbf{G}_0)$ of the player P and $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(\hat{g}(0) = \{0, z_0^{(1)}, z_0^{(2)},\cdots,z_0^{(n)}\} = \hat{g}_0 \in \hat{\mathbf{G}}_0)$ of the player E, which formed due from the τ-position $g(\tau)$ of the player P.

Then, on the basis of the above definitions, solving of the Problem 1, and results of the works [3], [4] the procedure of the construction the solution of the Problem 2 for the discrete-time dynamical process (1)-(8) can be represented as a sequence consisting from solving of the following three sub-problems:

1) constructing of the reachable set $\mathbf{Y}(\tau, y(\tau), u(\cdot), v(\cdot), Y_1(\cdot), T)$ (note, that this set can be constructed by finding a solutions of a finite sequence a linear mathematical programming problems, and this set is convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbb{R}^r$ [3], [4]);

2) for every fixed collections $(u(\cdot), v(\cdot)) \in \mathbf{U}(\overline{\tau, T}) \times \mathbf{V}(\overline{\tau, T}; u(\cdot))$ maximizing of the linear terminal functional $\hat{\alpha}$ which is defined by the relations (10), (11) on the sets $\mathbf{Y}(\tau, y(\tau), u(\cdot), v(\cdot), Y_1(\cdot), T)$ and $\mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T)$, $i \in \overline{1, n}$, $(v(\cdot) = (v_1(\cdot), v_2(\cdot), \cdots, v_n(\cdot)))$ namely, the formation of the following number:

$$\lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), u(\cdot), v(\cdot)) = \mu \cdot \hat{\gamma}(y^{(e)}(T)) + \sum_{i=1}^{n} \mu^{(i)} \cdot \hat{\beta}^{(i)}(z^{(i,e)}(T)) = \mu \cdot < e, y^{(e)}(T) >_r + \sum_{i=1}^{n} \mu^{(i)} \cdot < e^{(i)}, z^{(i,e)}(T) >_{s_i}$$

$$= \max_{y(T) \in \mathbf{Y}(\tau, y(\tau), u(\cdot), v(\cdot), Y_1(\cdot), T)} \mu < e, y(T) >_r + \sum_{i=1}^{n} \max_{z^{(i)}(T) \in \mathbf{Z}^{(i)}(\tau, z^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), Z_1^{(i)}(\cdot), T)} \mu^{(i)} \cdot < e^{(i)}, z^{(i)}(T) >_{s_i} \tag{23}$$

(note, that the solution of this problem is reduced to solving $(n+1)$ a linear mathematical programming problems [3], [4]);

3) constructing of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, g(\tau))$ and the number $\tilde{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau))$ from solving the following optimization problem:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, g(\tau)) = \{\tilde{u}^{(e)}(\cdot) : \ \tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T}), \ \tilde{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau)) = \lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot))$$

$$= \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), \tilde{u}^{(e)}(\cdot))} \lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) = \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, T})} \ \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u(\cdot))} \lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot))\} \tag{24}$$

(note, that the set $\mathbf{U}(\overline{\tau, T}; u(\cdot))$ is a finite set at the space $\mathbb{R}^p$, and the finite set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u(\cdot))$ constructed from (20), and then the solution of this problem is reduced to solving the finite discrete optimization problem).

Taking into consideration (9)-(11), (15), (20)-(24), and the conditions stipulated for the system (1)-(8), one can prove (analogy as in works [3], [4]), that the following assertion is valid.

Theorem 2. For fixed time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible on the levels I and II of the two level hierarchical control system for the discrete-time dynamical process (1)-(8) realizations τ-positions $g(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \mathbf{G}(\tau)$ $(g(0) = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = g_0 \in \mathbf{G}_0)$ of the player P and $\hat{g}(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(\hat{g}(0) = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = \hat{g}_0 \in \hat{\mathbf{G}}_0)$ of the player E, which formed due from the τ-position $g(\tau)$ of the player P, the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, g(\tau))$ of the admissible program controls $\tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of the control system and the number $\tilde{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau))$ constructed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problem, and the following equalities are true:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, g(\tau)) = \mathbf{U}^{(e)}(\overline{\tau, T}, g(\tau)); \ \tilde{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau)) = c_\alpha^{(e)}(\overline{\tau, T}, g(\tau)), \tag{25}$$

where set $\mathbf{U}^{(e)}(\overline{\tau, T}, g(\tau))$ is the set of minimax program controls of the player P on the level I of the control system for the dynamical process (1)-(8) and the number $c_\alpha^{(e)}(\overline{\tau, T}, w(\tau)))$ is the value of the result of the program minimax control for player P on the level I of this control system, and both satisfies the relation (15).

Then from this assertion follows that a solution of the Problem 2 for the discrete-time dynamical process (1)-(8) can be formed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problems on the basis of construction of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, g(\tau))$ and the number $\tilde{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau))$.

On the basis of the above algorithms of solving the Problems 1 and 2 the procedure of constructing the solution of the Problem 3 for the discrete-time dynamical system (1)-(8) can be represented as a sequence consisting from solving of the following two sub-problems:

1) for any control $\tilde{u}^{(e)}(\cdot) \in \tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, g(\tau))$ of the player P the constructing of the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), \tilde{u}^{(e)}(\cdot))$ and the number $\bar{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau))$ from solving the following optimization problem:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), \tilde{u}^{(e)}(\cdot)) = \{\bar{v}^{(e)}(\cdot) : \ \bar{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), \tilde{u}^{(e)}(\cdot)), \ \bar{c}_\alpha^{(e)}(\overline{\tau, T}, g(\tau)) = \lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), \tilde{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot))$$

$$= \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), \tilde{u}^{(e)}(\cdot))} \lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) = \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, T})} \ \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, \hat{g}(\tau), u(\cdot))} \lambda_\alpha^{(e)}(\overline{\tau, T}, g(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot)) \tag{26}$$

(note, that the sets $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),\tilde{u}^{(e)}(\cdot))$ and $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},g(\tau))$ constructed from relations (20) and (24), respectively, and then the solution of this problem is reduced to solving the finite discrete optimization problem);

2) for any control $\tilde{u}^{(e)}(\cdot)\in\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T})$ of the player P and any control $\bar{v}^{(i,e)}(\cdot)\in\bar{\mathbf{V}}^{(i,e)}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot))$ of the player E_i the constructing of the number $\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot))$ from solving the finite discrete optimization problem described by the relation (19) and satisfying the following relation:

$$\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot))=\kappa^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(i,e)}(\cdot))$$

$$=<e^{(i)},\bar{z}^{(i,e)}(T)>_{s-i}=\max_{z^{(i)}(T)\in\mathbf{Z}^{(i)}(\tau,z^{(i)}(\tau),\tilde{u}^{(e)}(\cdot),\bar{v}^{(i,e)}(\cdot),Z_1(\cdot),T)}<e^{(i)},z^{(i)}(T)>_{s_i}. \quad (27)$$

Taking into consideration (18)-(27), and the conditions stipulated for the discrete-time dynamical process (1)-(8), one can prove that the following assertion is valid.

Theorem 3. For fixed time interval $\overline{\tau,T}\subseteq\overline{0,T}$ $(\tau<T)$ and admissible on the level I of the two-level hierarchical control system for the discrete-time dynamical process (1)-(8) realization the τ-position $g(\tau)=\{\tau,y(\tau),z^{(1)}(\tau),z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\}\in\mathbf{G}(\tau)$ $(g(0)=\{0,y_0,z_0^{(1)},z_0^{(2)},\cdots,z_0^{(n)}\}=g_0\in\hat{\mathbf{W}}_0)$ of the player P and admissible on the level II of the control system for this dynamical process the realization τ-position $\hat{g}(\tau)\in\hat{\mathbf{G}}(\tau)$ $(\hat{g}(0)=\hat{g}_0\in\hat{\mathbf{G}}_0)$ of the player E which formed due from the τ-position $g(\tau)$ and admissible realization of the program minimax control $u^{(e)}(\cdot)\in\mathbf{U}^{(e)}(\overline{\tau,T},g(\tau))$ of the player P on the level I of the control system, which formed from solving the Problem 1 and Problem 2, the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))\subseteq\mathbf{V}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))\subseteq\mathbf{V}(\overline{\tau,T};u^{(e)}(\cdot))$ of the admissible program controls $\bar{v}^{(e)}(\cdot)\in\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))$ of the player E on the level II of the control system for this dynamical process and the number $\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(1)(\tau),u^{(e)}(\cdot))$ which form due from (26) and (27), respectively, constructed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problem, and the following equalities are true:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))=\hat{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot));\ \bar{c}^{(e)}_{\alpha}(\overline{\tau,T},g(\tau))=c^{(e)}_{\alpha}(\overline{\tau,T},g(\tau));$$

$$\forall\ i\in\overline{1,n}:\ \bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),u^{(e)}(\cdot))=c^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),u^{(e)}(\cdot)). \quad (28)$$

Then from this assertion follows that a solution of the Problem 3 for the discrete-time dynamical process (1)-(8) can be formed from a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problems on the basis of construction of the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},\hat{g}(\tau),u^{(e)}(\cdot))$ and numbers $\bar{c}^{(e)}_{\alpha}(\overline{\tau,T},g(\tau))$ and $\bar{c}^{(i,e)}_{\beta_i}(\overline{\tau,T},g^{(i)}(\tau),u^{(e)}(\cdot))$.

Note, that on the basis of the above algorithms of solving the Problems 1-3 the procedure – common algorithm of the construction a solution of the main problem of two-level hierarchical minimax program control by the final phase states of the objects I and II_i, $i\in\overline{1,n}$ for the discrete-time dynamical process (1)-(8) in the presence of perturbations can be formed from realization of a finite number procedures of solving the linear mathematical programming problems and the finite discrete optimization problems.

CONCLUSION

In conclusion we note that more concrete algorithm for realization of the minimax program terminal control by the final phases state of regional social and economic system in the presence of risks for the discrete-time dynamical process (1)-(8) can be constructed using algorithms for solving minimax program terminal control problems with incomplete information from works [3], [4].

Results of this paper can be used for computer simulation, design and construction of multilevel control systems for actual economic, technical and other dynamical processes operating under deficit of information and uncertainty.

ACKNOWLEDGEMENTS

This work was supported by the Russian Science Foundation, project no. 15-18-10014, “Projection of optimal socio-economic systems in turbulence of external and internal environment.”

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